

PART 2

# Algebra Topics

(1) Evaluate each expression. Be sure to show your work.

a) If  $x=5$ ,  $y=-2$ , and  $w=6$  find the value of

$$\frac{4x-3y+3w}{2xw-5y}$$

b) If  $x=4$ ,  $y=3$ , and  $w=-2$ , find the value of

$$\frac{(2x-3w)^2+2y^3}{x^2-y^2}$$

(2) Solve each equation for "x". Show all necessary steps.

a)  $2x + 4(3-5x) + 10 = 9x + 11$

b)  $\frac{5x}{7} + \frac{3x}{2} = \frac{4+11x}{14}$

c)  $6x + 4(2x-3) = 4[3(2x-1)] + 10x$

(3) Evaluate  $f(x) = 3x^2 - 2x + 7$  for

$f(2)$

$f(0)$

$f(-7)$

$f(2b)$

(4) Given  $f(x) = 2x + 1$ ,  $g(x) = 5 - 3x$ ,  $h(x) = 3x^2 + x - 2$  find

(a)  $f(x) + h(x)$

(b)  $h(x) - g(x)$

(c)  $f(x) \cdot g(x)$

(d)  $\frac{h(x)}{f(x)}$

(e)  $f(x) - g(x) + 2h(x)$

(f)  $g(h(x))$

(g)  $f(g(x))$

(5) a) Find the equation for the inverse function of  $f(x) = 7x - 8$

b) Find the equation for the inverse function of  $f(x) = 2x^2 + 6$ .

# 1) Quadratic Functions and Equations

Solve each of the following quadratic equations by any method. Show all steps.

(a)  $x^2 - 10x - 21 = 0$

(b)  $3x^2 - 2x - 1 = 0$

(c)  $5x^2 + 6x - 9 = 0$

# 2) Graphing quadratic functions

Graph each of the following on the coordinate graph paper provided. Show each of the five steps needed to find the coordinates for your graph.

(a)  $y = x^2 - 8x + 12$

(b)  $y = 4x^2 + 4x - 15$

(c)  $y = x^2 - 4x + 3$

(d)  $y = x^2 + 5x + 24$

3) a) Find the equation of a parabola with x-intercepts 2 and -1 and y-intercept 6.

b) Find the equation of a parabola with x-intercepts 5 and 1 and y-intercept = 1.

# Systems of Equations:

Find the Point of Intersection for Each System of Equations.

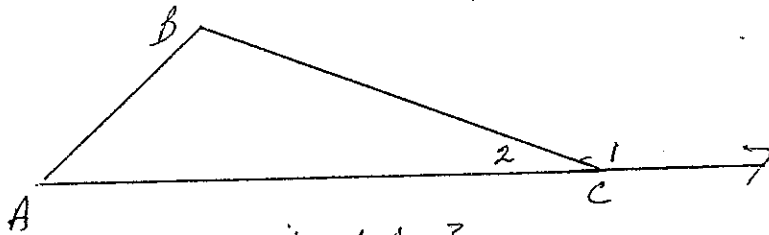
(1)  $3x + 7y = 10$   
 $5x - 2y = 3$

(2)  $6x - 5y = 11$   
 $-7x + 2y = -22$

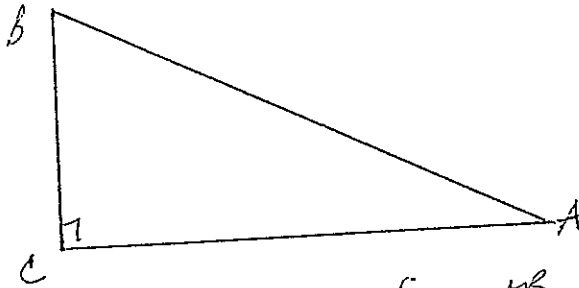
(3)  $8x + y = 4$   
 $9x - 2y = 17$

} use the substitution method

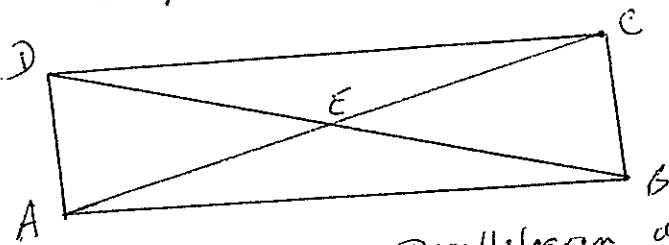
# GEOMETRY AND TRIGONOMETRY



- 1) Which side is opposite  $\angle A$ ?
- 2) Which angle is opposite  $\overline{AB}$ ?
- 3)  $m\angle A + m\angle B + m\angle C = \underline{\hspace{2cm}}$ ?
- 4)  $m\angle 1 = m\text{---} + m\text{---}$ .
- 5) If  $m\angle 2 = 23$  and  $m\angle B = 105$ ,  $m\angle A = \text{---}$ .
- 6)  $m\angle 2 = 2x + 3$ ,  $m\angle B = 5x + 8$ ,  $m\angle A = 3x - 1$ . Find  $m\angle A$ ,  $m\angle B$ , and  $m\angle C$ .
- 7) Which of the following could not be lengths of a triangle?  
 (a) 5, 6, 7      (b) 2, 3, 10      (c) 7, 7, 7      (d) 3, 4, 5



- 1) If  $BC = 5$  and  $AC = 12$ , find  $AB$ . (Show work.)
- 2) If  $AB = 17$  and  $AC = 15$ , find  $BC$ . (Show work.)
- 3) If  $m\angle A = 30$  and  $AB = 16$ , find  $BC$  and  $AC$ .
- 4) If  $\triangle ABC$  is an isosceles right triangle with  $AC = BC$  (use your imagination), find  $AB$  if  $BC = 11$ .

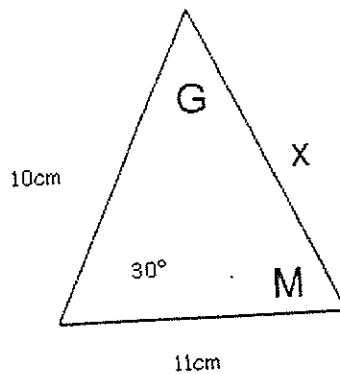
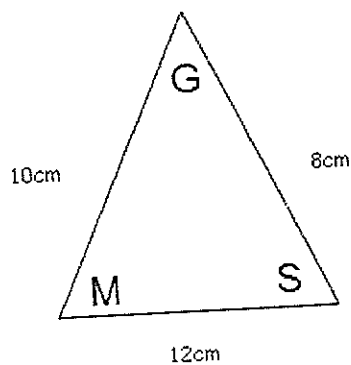


- 1) If  $ABCD$  is a parallelogram and  $DB = 21$ ,  $DE = \text{---}$ ?
- 2) If  $AE = 10.7$ ,  $EC = \text{---}$ ?
- 3) If  $m\angle AEB = 123$ ,  $m\angle DEC = \text{---}$ ?
- 4) If  $ABED$  is a rectangle,  $AC = 7x + 5$  and  $DB = 9x - 16$ , find  $AC$ ,  $DB$ , and  $AE$ . (Show work.)
- 5) If  $ABCD$  is a rectangle,  $AD = 7$  and  $AB = 24$ , find  $DB$ . (Show work.)

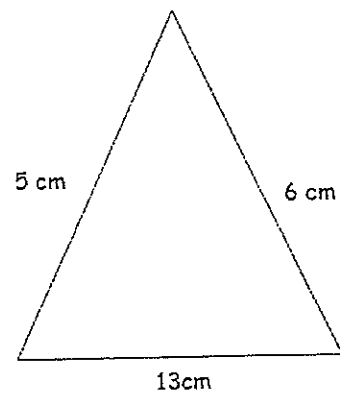
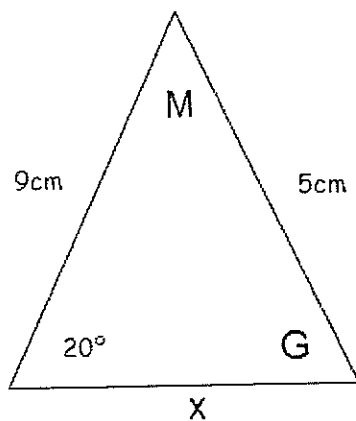
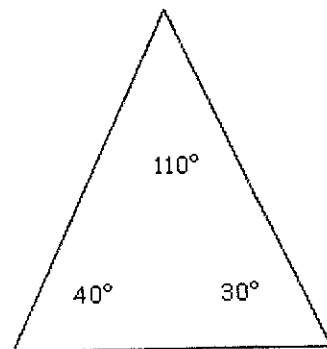
### Exercises

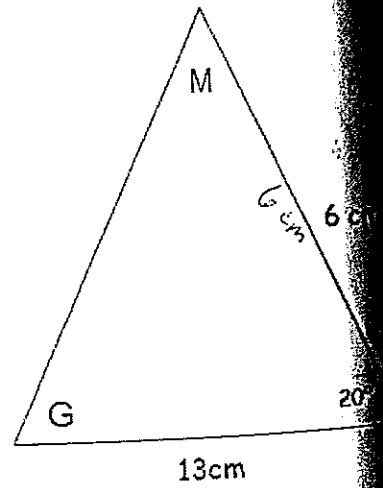
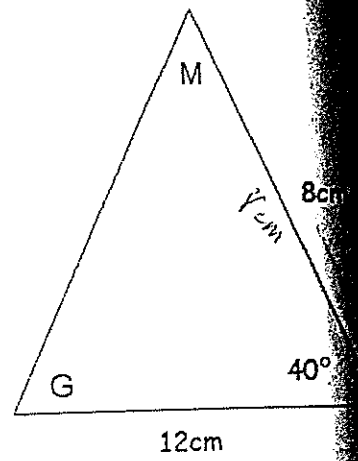
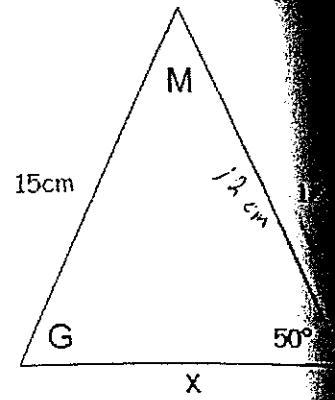
1. Sketch a right angled triangle with angles: M, N, G and sides x, y, z. Write the Sine and Cosine rule for this triangle.

2. Find all the missing sides, angles and area of the triangles below. If there is more than one set of solutions, try to find them all.



4. Find all the sides, angles and the area of the following triangles:





### 6.3. - TRIGONOMETRIC RATIOS

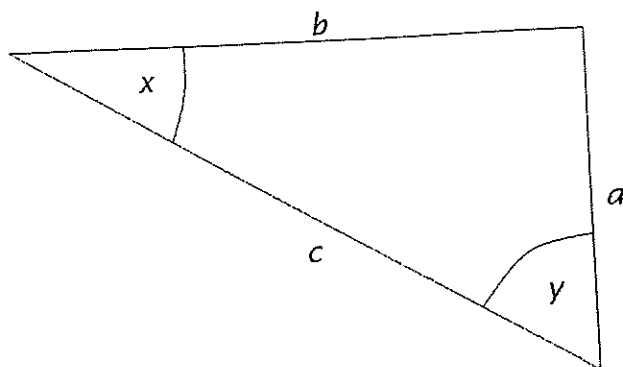
Following directly from the unit circle are the trigonometric ratios:

$$\sin(x) = \frac{a}{c} = \cos(y)$$

$$\cos(x) = \frac{b}{c} = \sin(y)$$

$$\tan(x) = \frac{a}{b} = \frac{\sin(x)}{\cos(x)}$$

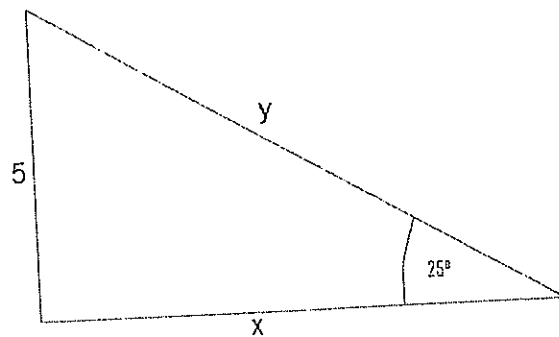
$$\tan(y) = \frac{b}{a} = \frac{\sin(y)}{\cos(y)}$$



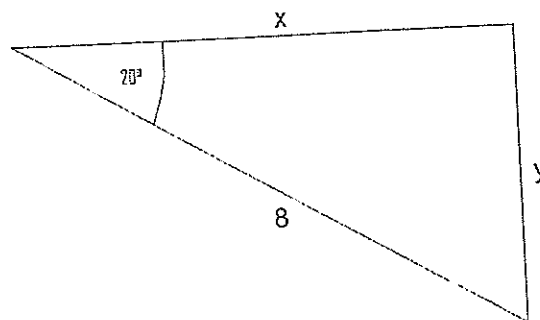
Exercises:

1. Find  $x$  and  $y$  in the following cases:

a.

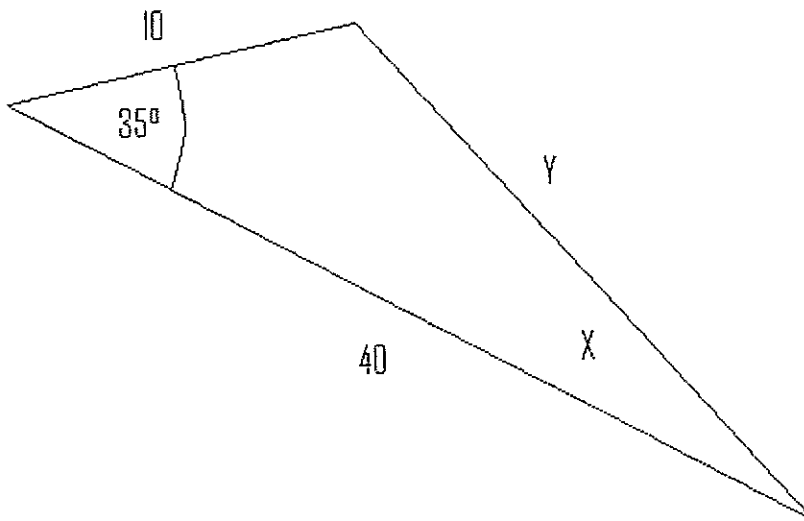


b.



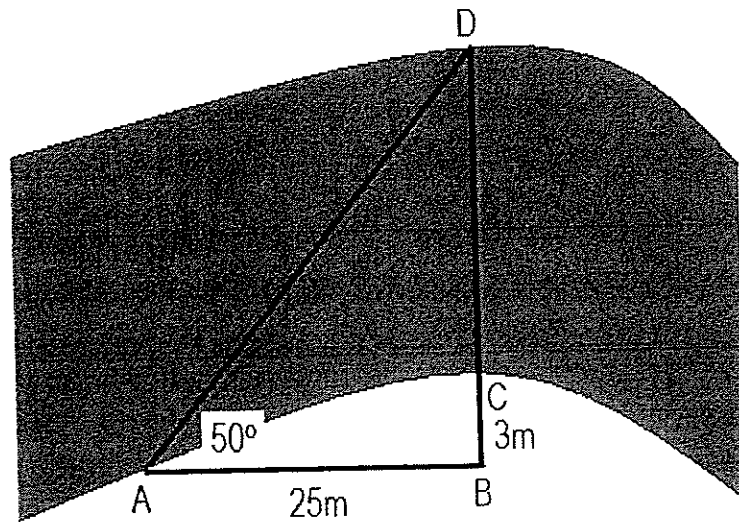


2. The Triangle in the diagram (not to scale) is not right angled, find  $x$  and  $y$ .



3. The shade formed by building is 100m long. The depression angle of the light as it approaches the ground is  $40^\circ$ .
- Sketch a diagram that describes the situation.
  - Find the height of the building.

4. John who lives next the river wanted to measure its width without crossing the river. He did some measurements and obtained the following data:



$$AB = 25\text{m}, BC = 3\text{m}, \angle B = 90^\circ$$

Find the width of the river.

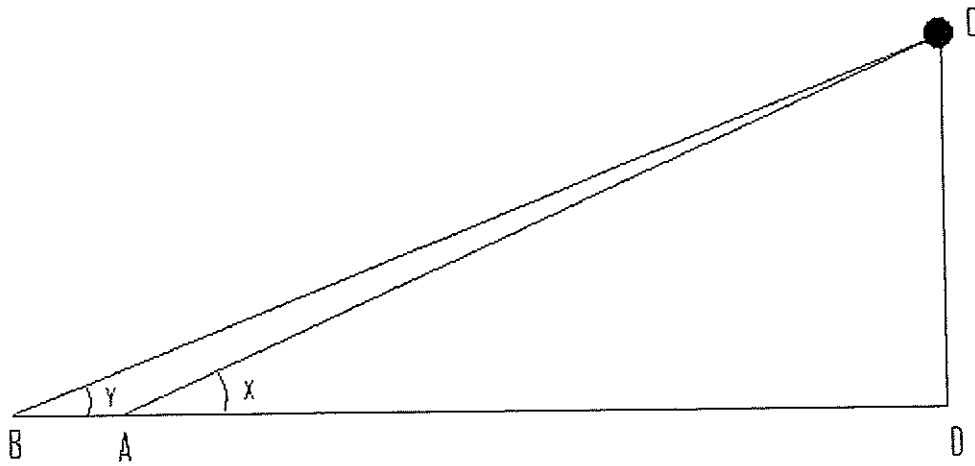
5. The height of building is 120m. The depression angle of the light as it approaches the ground is  $30^\circ$ .

- a. Sketch a diagram that describes the situation.
- b. Find the length of the shade on the ground.

6. In its search for food the lion is observing a certain prey located 2 m above the ground. The lion's head forms an angle of  $12^\circ$  with the ground as he looks at his prey.

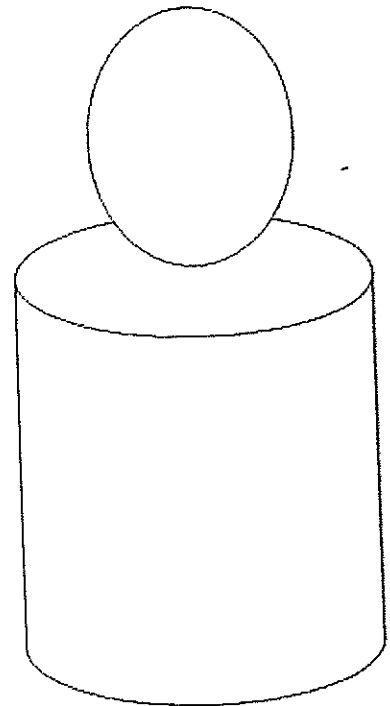
- a. Sketch a diagram that describes the situation.
- b. Find the distance from the lion's mouth to its prey.

7. Measuring the height and distance of objects:

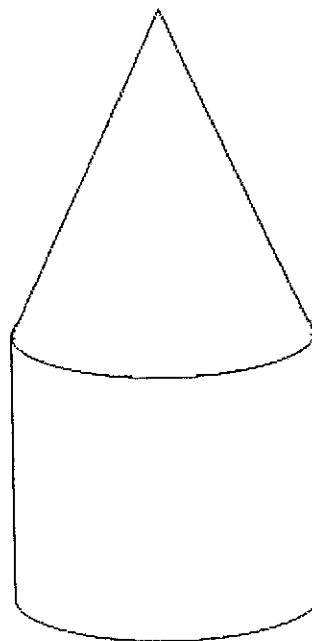


Assuming we start at point A and the object that we want to measure its distance (ground distance AD or Air distance AC) is located at point C. We need to use a device (in real life a **theodolite** is used) to measure the angle  $x$  (elevation angle), let's assume that we measured it and got  $20^\circ$ . Later we move a certain distance (backwards or forward) and measure the angle  $y$ . Let assume that we moved backwards 4 meters (that is  $AB = 4\text{m}$ ) and the angle  $y$  is  $18^\circ$ . Find AD, AC, CD.

3. In the design process of a modern building a sphere of 5m radius is put on top a cylinder with a radius twice as big. The height of the building is 30m.
- Find the volume of the sphere.
  - Find the height of the cylinder.
  - Find the volume of the building.
  - Find the surface area of the building that that is exposed to fresh air.

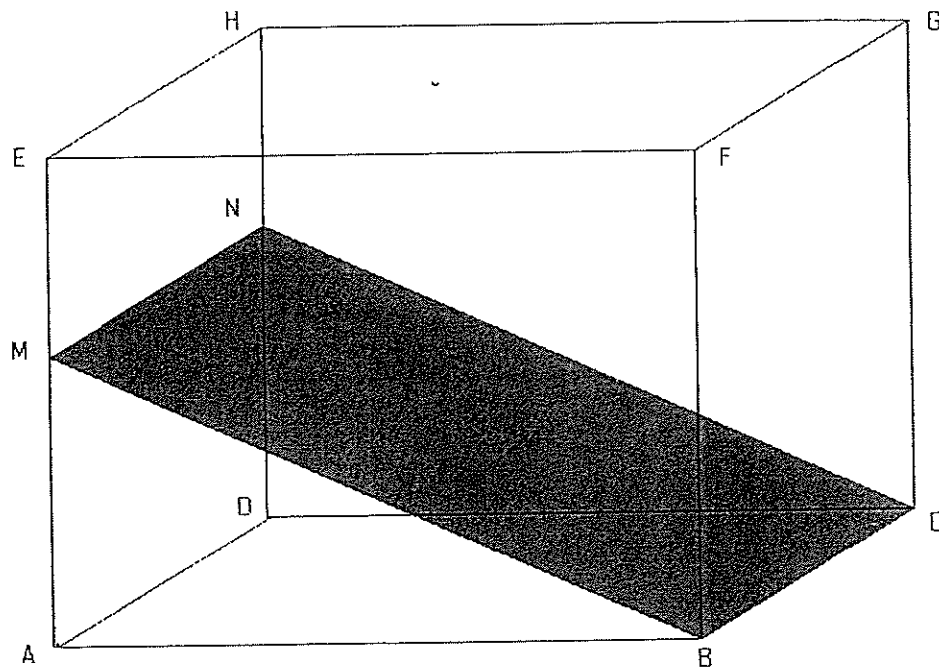


5. An old tower is made of a cone put on top of a cylinder. The radius of both is 5m. The height of the cylinder is 10m. The height of the cone is 60% of the cylinder's height.
- Find the height of the tower.
  - Find the volume of the tower.



3. Given the following diagram (not to scale): ABCD is a rectangle  $AB = 20$  cm,  $BC = 12$  cm,  $EA = BF = 14$  cm.  $EM = 5$  cm.

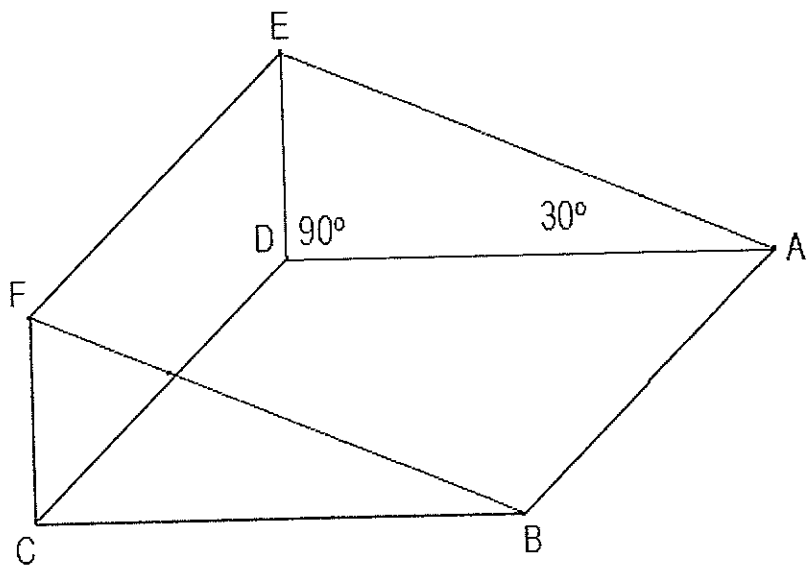
- Find the angle between NB and the base ABCD.
- Find the length of the segment MC.
- Find the area of MNBC
- Find the volume of the cuboid.
- Find the surface area of the cuboid.



4. Given the following right prism.  $AB = 12\text{cm}$ ,  $AE = 15\text{cm}$ .

Find:

- The length of  $AD$ .
- The length of  $ED$ .
- The length of  $AF$ .
- The angle  $FAB$ .
- The surface area of the prism.





## COORDINATE GEOMETRY

FOR THE GIVEN YOU ARE GIVEN COORDINATES OF THE ENDPOINTS OF TWO SEGMENTS A and B in line  $l_3$  and C and D in line  $l_4$

FIND THE FOLLOWING - SHOW ALL WORK

$$l_3 \quad A = (3, -4) \quad B = (7, 3) \quad l_4 \quad C = (-1, -8), \quad D = (3, 5)$$

- GRAPH  $l_3$  and  $l_4$
- FIND THE SLOPE OF  $l_3$  and THE SLOPE OF  $l_4$
- FIND THE LENGTH OF  $\overline{AB}$  and OF  $\overline{CD}$ .
- FIND THE COORDINATE OF THE MIDPOINT OF  $\overline{AB}$  and THE COORDINATE OF THE MIDPOINT OF  $\overline{CD}$ .
- FIND THE EQUATION OF  $l_3$  and the EQUATION of  $l_4$
- FIND THE SLOPES OF LINES PARALLEL TO  $l_3$  and  $l_4$ .
- FIND THE SLOPES OF LINES PERPENDICULAR TO  $l_3$  and  $l_4$ .
- FIND THE POINT OF INTERSECTION OF  $l_3$  and  $l_4$ .

Write a 2 - 3 pages report on Rene Descartes and the role that he played in the founding of coordinate geometry.

### 3.2. - LOGIC

1. Logic is a way to describe situations or knowledge in a way that allows drawing new conclusions. It is useful in computer programming, artificial intelligence and better understanding of language.

2. For example:

- a. All swimmers can swim fast
- b. Daniel is a swimmer

Therefore by using logic, Daniel can \_\_\_\_\_

3.  $p$ : A proposition that is either true or false

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example that is not a proposition: \_\_\_\_\_

Example that is not a proposition: \_\_\_\_\_

4.  $\neg p$           Negation of  $p$

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example: \_\_\_\_\_

5. Compound statements: A compound statement is: \_\_\_\_\_

\_\_\_\_\_

6.  $q \wedge p$           Conjunction:  $q$  and  $p$

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example: \_\_\_\_\_

7.  $q \vee p$  Disjunction: q or p

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example: \_\_\_\_\_

8.  $q \vee p$  Exclusive Disjunction: q or p but not both

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example: \_\_\_\_\_

9.  $p \Rightarrow q$  Implication

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example: \_\_\_\_\_

10.  $p \Leftarrow q$  Converse of an Implication

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example: \_\_\_\_\_

11.  $\neg p \Rightarrow \neg q$  Inverse of an Implication

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example: \_\_\_\_\_

12.  $\neg p \Leftarrow \neg q$       Contrapositive of an Implication

Example: \_\_\_\_\_

Example: \_\_\_\_\_

Example: \_\_\_\_\_

13.  $p \Leftrightarrow q$       Equivalence of p and q

Two statements p, q are equivalent if \_\_\_\_\_

If  $p \Rightarrow q$  and  $p \Leftarrow q$  are both \_\_\_\_\_ we can say that \_\_\_\_\_

Example:

14. Valid arguments: An argument is valid if the conclusion follows the statements, even if the statements are incorrect. For example:

- a. Oranges are white
- b. White fruit are sweet

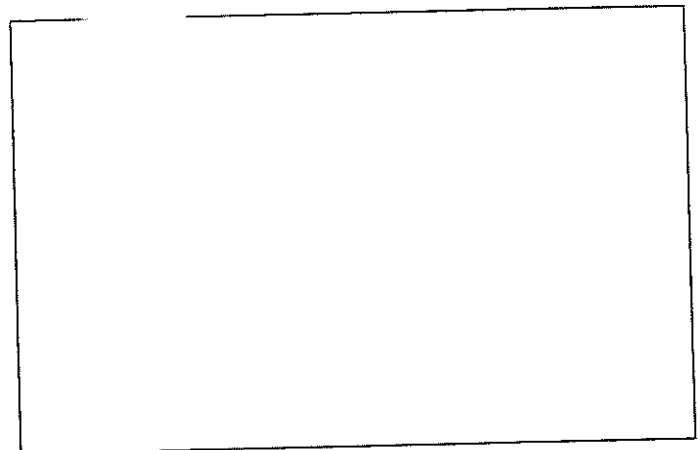
Therefore by logic, \_\_\_\_\_

15. There is a deep analogy between set theory and logic. For example:

Disjoint: A red chair is never blue, that can be observed in the following sets exercise:

Sketch the corresponding Venn diagram to the following sets:

- A: Set of red chairs
- B: Set of blue chairs
- C: Set of red chairs with wheels
- D: Set of high chairs



### 3.1. – SET THEORY

1. A set is \_\_\_\_\_.
2. Give 3 examples of sets:
3. Consider the set  $\{2, 4, 6, \dots\}$ 
  - a. This is the set of \_\_\_\_\_. The next element is \_\_\_\_
  - b. In this set the number of elements is \_\_\_\_ It is an \_\_\_\_\_ set
4. Consider the set  $\{1, 8, 27, \dots\}$ 
  - a. This is the set of \_\_\_\_\_. The next element is \_\_\_\_
  - b. In this set the number of elements is \_\_\_\_ It is an \_\_\_\_\_ set
5. Consider the set  $\{\text{Asia, Africa, } \dots\}$ 
  - a. This is the set of \_\_\_\_\_. The next element is \_\_\_\_
  - b. In this set the number of elements is \_\_\_\_ It is an \_\_\_\_\_ set

6. A **subset** is \_\_\_\_\_. It is denoted by  $A \subseteq B$

7. Given the set  $L = \{A, B, C\}$

a. State all the possible subsets of L. include the empty set.

L1 = \_\_\_\_\_

L2 = \_\_\_\_\_

L3 = \_\_\_\_\_

L4 = \_\_\_\_\_

L5 = \_\_\_\_\_

L6 = \_\_\_\_\_

L7 = \_\_\_\_\_

L8 = \_\_\_\_\_

b. All the subsets except \_\_\_\_\_ are called **proper subsets**  
**denoted by**  $A \subset B$

c. Explain the difference between a subset and a proper subset.

d.  $A \not\subset B$  means \_\_\_\_\_

e. \_\_\_\_\_ means that A is NOT a subset of B

8. M is the set of perfect square smaller than a 100.

a. List the elements of M \_\_\_\_\_

b. List the subset Q of even numbers in M \_\_\_\_\_

9. N is the set of prime numbers between 10 and 30.

a. List the elements of M \_\_\_\_\_

b. List the subset Q of even numbers in M \_\_\_\_\_

10. The **universal set** is particular for \_\_\_\_\_ and contains \_\_\_\_\_ for the problem. Usually it is denoted by the letter \_\_\_\_\_.

11. The universal set for the students in the classroom is

$U =$  \_\_\_\_\_

12. Given the sets  $U = \{\text{John, Raquel, Felix, Shan, Mila, Jessy, Pamela}\}$  and the subset of  $U$ :  $B = \{\text{Shan, Mila}\}$ .

State the complement of the set  $B' =$  \_\_\_\_\_

13. The **complement of a set** \_\_\_\_\_

14. The **intersection** of 2 sets is \_\_\_\_\_. It is denoted by  $A \cap B$ .

15. The **union** of 2 sets is \_\_\_\_\_. It is denoted by  $A \cup B$

16. For example if  $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$  and  $M = \{2, 6, 10, 12\}$

a.  $S \cap M =$  \_\_\_\_\_

b.  $S \cup M =$  \_\_\_\_\_

17. Given the sets  $U = \{\text{John, Raquel, Felix, Shan, Mila, Jessy, Pamela}\}$  and the subset of  $U$ :  $B = \{\text{Shan, Mila}\}$ .

a.  $U \cap B =$  \_\_\_\_\_

b.  $U \cup B =$  \_\_\_\_\_



## Venn diagrams

Event	Set Language	Venn diagram	Probability result
Complementary event ( $A'$ )	Not A		$P(A') =$
The _____ of A and B $(A \cap B)$	Set of elements that belongs to A _____ B		$P(A \cup B) =$
The _____ of A and B $(A \cup B)$	Set of elements that belongs to A _____ B ____ both		
If $(A \cap B) = \emptyset$ A and B are said to be:	The sets A and B are _____ _____		$P(A \cup B) =$ $P(A \cap B) =$

## Exercises

1. The events A and B are such  $P(A) = 0.2$ ,  $P(B) = 0.4$  and  $P(A \cup B) = 0.5$ . Find:
  - a.  $P(A \cap B)$
  - b.  $P(B')$
  - c. Sketch the corresponding Venn diagram.
  - d.  $P(A' \cap B)$
  - e.  $P(A' \cap B')$
  - f. Are the events A and B Independent? Explain.

2. The events A and B are such  $P(A) = 0.15$ ,  $P(B) = 0.3$  and  $P(A \cup B) = 0.4$ ,  
Find:

- a.  $P(A \cap B)$
- b.  $P(B')$
- c. Sketch the corresponding Venn diagram.
- d.  $P(A' \cap B)$
- e.  $P(A' \cap B')$
- f. Are the events A and B Independent? Explain.

3. The events A and B are such  $P(A) = 0.3$ ,  $P(B) = 0.6$  and  $P(A \cup B) = 0.9$ , Find:

- a.  $P(A \cap B)$
- b.  $P(B')$
- c. Sketch the corresponding Venn diagram.
- d.  $P(A' \cap B)$
- e.  $P(A' \cap B')$
- f. Are the events A and B Independent? Explain.

4. The events A and B are such  $P(A) = 0.2$ ,  $P(B) = 0.9$  and  $P(A \cap B) = 0.1$ , Find:

- a.  $P(A \cup B)$
- b.  $P(B')$
- c. Sketch the corresponding Venn diagram.
- d.  $P(A' \cap B)$
- e.  $P(A' \cap B')$
- f. Are the events A and B Independent? Explain.

5. 20% of certain city census consume alcohol regularly, 40% do sport regularly and 10% do both.
- Represent the information in a diagram.
  - Calculate the probability that someone chosen at random only drinks alcohol regularly.
  - Calculate the probability that someone chosen at random only drink alcohol regularly or only practices sport regularly (but not both).
  - Calculate the probability that someone picked at random does not drink alcohol nor practices sport regularly.

6.  $P(A) = 0.46$ ,  $P(B) = 0.33$ ,  $P(A \cap B) = 0.15$ .

- Represent the information in a diagram.
- Find the probability that an event is not A nor B.

## CONDITIONAL PROBABILITY

Informal definition: **Knowing** that B has happened, what is the probability that A will happen (Written as  $P(A|B)$ )

Formal definition: The probability of and event A given event B is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

## INDEPENDENT EVENTS

Informal definition:  $P(B)$  is not influenced by  $P(A)$ .

Formal definition :  $P(A \cap B) = P(A)P(B)$

### Exercises

1. What is the difference between independent events and mutually exclusive events?
  
2. Give an example of independent events.
  
3. In a certain town the probability of a rainy day is 0.58 and the probability of strong wind is 0.76. If these are independent events, find the probability of:
  - a. A rainy windy day.
  - b. A dry windy day.
  - c. A dry and not windy day.
  - d. 2 consecutive rainy days.
  - e. 2 consecutive windy rainy days.

## Lattice diagrams

4. Two dice numbered one to six are rolled onto a table.
  - a. Sketch a corresponding diagram.
  - b. Find the probability that the sum is 7.
  - c. Find the probability that the sum is more than 7.
  - d. Find the probability that the sum is less than 4.
  - e. Find the probability that the sum is even.
  - f. Find the probability of obtaining a sum of five given that the sum is seven or less.
  - g. Find the probability of obtaining a sum of 4 given that the sum is even.

5. A die and coin are rolled on a table.
  - a. Sketch a corresponding diagram.
  - b. Find the probability of getting Tail and an even number.
  - c. Find the probability of getting Tail and a 4.
  - d. Find the probability of obtaining a 5 knowing that a tail was obtained.

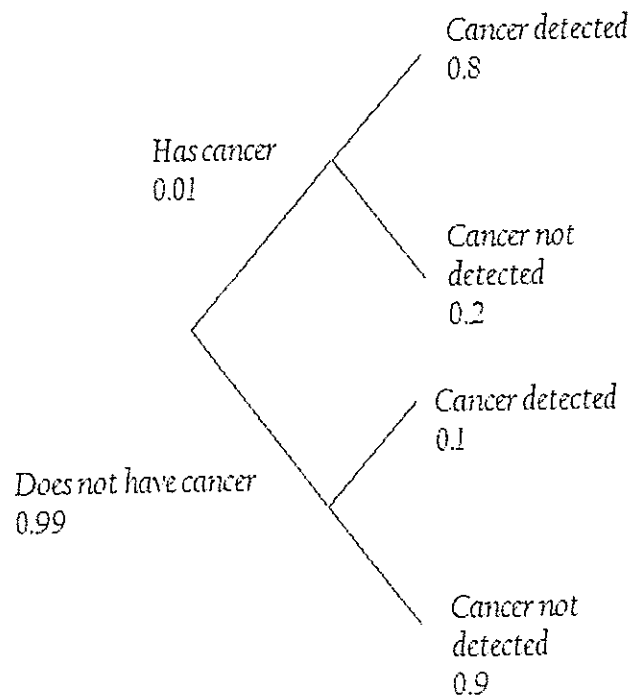
## TOTAL PROBABILITY

### Solved Example

It is known that:

- i. 1% of women aged 40 have breast cancer
- ii. A mammography test has 80% success rate.
- iii. A mammography test has 10% false alarm rate

A woman receives a positive mammography test, what is the probability she really has cancer?



$$P(\text{True yes} | (\text{All Yeses})) = \frac{0.01 \cdot 0.8}{0.01 \cdot 0.8 + 0.99 \cdot 0.1} \approx 0.0748$$

The woman has 7.5% probability to have cancer.



## TREE DIAGRAMS

6. If the probability of tail is 0.53, find the probability of at least one tail in 2 throws.
7. An urn contains 8 cubes of which 5 are black and the rest are white.
- What is the probability to draw a white cube?
  - Draw a tree diagram in case a 1<sup>st</sup> cube is drawn, it is **NOT replaced** and then another cube is drawn. Indicate all the probabilities on the tree diagram.
  - Calculate the probability to draw 2 consecutive black cubes.
  - Calculate the probability to draw **at least** 1 black cube.
  - Given that the first cube drawn was white, calculate the probability that the 2<sup>nd</sup> is black.

8. A bag contains 3 red balls, 4 blue balls and 5 green balls. A ball is chosen at random from the bag and is not replaced. A second ball is chosen. Find the probability of choosing one green ball and one blue ball in any order.

9. Given that events  $A$  and  $B$  are independent with  $P(A \cap B) = 0.4$  and  $P(A \cap B^c) = 0$ . Find  $P(A \cup B)$ .

10. Given that  $P(A) = 0.4$ ,  $P(B) = 0.7$  and  $P(A \cup B) = 0.8$ . Find:

- a.  $P(A \cap B)$
- b.  $P(A | B)$
- c. Determine if A and B are independent events.

11. Given that  $P(A) = 0.4$ ,  $P(B) = 0.6$  and  $P(A \cup B) = 0.76$ .

- a. Find  $P(A \cap B)$
- b. Are events A and B mutually exclusive? Explain.
- c. Are events A and B independent?

